Algebra II

8-6

Some Useful Theorems

Theorem - A polynomial function $P(x)$ with degree n has exactly n roots.

x-intercepts
zeros
solutions.

How many solutions do each of the following have?

$P(x) = x^3 + 5x^2 - 7x + 1$ 3

$P(x) = x^{12} - 3x^4 + 8x$ 12

$P(x) = 4 + 3x^5 - 7x^6 + 11x^9 - 131x^2$ 9

The biggest power (degree term) gives the number of solutions.

Conjugate Root Theorem - If $P(x)$ has all real coefficients, then any complex solutions must come as conjugate pairs.

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All but one of the equations solutions are given. Find the remaining root.

5) $x^3 - 3x^2 + 4x - 12$ $\{3, 2i, -2i\}$

Imaginary solutions come as conjugate pairs (theorem)

Find a cubic equation with integral coefficients that has the given roots.

1) $\{-1, 5i, -5i\}$

$(x+1)(x-5i)(x+5i)=0$

$(x+1)(x^2-25i^2)=0$

$(x+1)(x^2+25)=0$

$\boxed{x^3+x^2+25x+25=0}$

Imaginary solutions come as conjugate pairs (theorem)

A root of the equation is given. Solve the equation.

9) $x^3 + x - 10 = 0$ $\{-1+2i, -1-2i, 2\}$

	1	0	1	-10
$-1+2i$		$-1+2i$	$-3-4i$	10
	1	$(-1+2i)$	$(-2-4i)$	0
$-1-2i$		$-1-2i$	$2+4i$	
	1	-2	0	

$x-2=0$

$x=2$

$(-1+2i)(-1+2i)$
$1-2i-2i+4i^2$
$-3-4i$

$(-2-4i)(-1+2i)$
$2-4i+4i-8i^2$
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Descartes' Rule of Signs -

*1) $3x^5 - 4x^3 - 7x^2 + 11x + 9 = 0$

1 2

For negatives, change the odd terms

$-3x^5+4x^3-7x^2-11x+9=0$

1 2 3

+	-	i
2	3	0
0	3	2
2	1	2
0	1	4

Without seeing the lecture, perhaps read the description in the textbook to help the understanding.

*2) $6x^6 + 7x^5 - x^4 + 2x^2 - x - 1 = 0$

1 2 3

$6x^6-7x^5-x^4+2x^2+x-1=0$

1 2 3

+	-	i
3	3	0
1	3	2
3	1	2
1	1	4

List all the possibilities for the nature of the roots of each equation.

13) $x^4 + 3x^2 - 4 = 0$

1

$x^4+3x^2-4=0$

1

+	-	i
1	1	2

19) $x^5 - x^3 - x^2 + x - 2 = 0$

1 2 3

$-x^5+x^3-x^2-x-2=0$

1 2

+	-	i
3	2	0
3	0	2
1	2	2
1	0	4

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